

STAT 511: Summer 2018 – Quiz 2

Name:

Class Time:

Let Z be a standard normal variable and let $X=Z^2$.

1. (1 point) What is the support of X ?

$$(0, \infty)$$

2. (2 points) Find $P(X < 2)$. Remember that Z can take all real values.

$$\begin{aligned} P(X < 2) &= P(Z^2 < 2) = P(-\sqrt{2} < Z < \sqrt{2}) = 2\Phi(\sqrt{2}) - 1 \\ &= 2\Phi(1.41) - 1 \\ &= 2 \times 0.9207 - 1 \\ &= 0.8414 \end{aligned}$$

3. (2 points) Using ideas from question 2, find a general form for the CDF of X involving the CDF $\Phi(z)$ of Z .

$$F_X(x) = P(X \leq x) = \begin{cases} P(Z^2 \leq x) = P(-\sqrt{x} \leq Z \leq \sqrt{x}) \\ \quad = 2\Phi(\sqrt{x}) - 1, \text{ if } x > 0 \\ 0, \text{ if } x \leq 0 \end{cases}$$

4. (3 points) Find the median of X .

Solve: $F_X(x) = 0.5 \quad (x > 0)$

$$\Rightarrow 2\Phi(\sqrt{x}) - 1 = 0.5$$

$$\Rightarrow \Phi(\sqrt{x}) = 0.75$$

$$\Rightarrow \sqrt{x} = 0.67 \Rightarrow x = (0.67)^2 = 0.4489$$

i.e. $\tilde{\mu}_X = 0.4489$
(From software, $\tilde{\mu}_X = 0.4549$)

5. (2 points) Find $E(X)$. You don't need to do any integration.

$$E(X) = E(Z^2) = V(Z) \quad (\text{as } E(Z) = 0) \\ = 1$$

6. (Bonus: 2 points) Find the PDF of X . [Hint: $\Phi'(z) = \phi(z)$]

$$f_X(x) = F_X'(x) = \begin{cases} 0, \text{ if } x \leq 0 \\ 2\Phi'(\sqrt{x}) \frac{d}{dx}(\sqrt{x}) = 2\phi(\sqrt{x}) \frac{1}{2} x^{-1/2} \\ \quad = \frac{\phi(\sqrt{x})}{\sqrt{x}}, \text{ if } x > 0 \\ \quad = \frac{1}{\sqrt{2\pi x}} e^{-\frac{x}{2}}, \text{ if } x > 0 \end{cases}$$

This is the 'Chi-Squared' (χ^2) Distribution with 1 degree of freedom.